Modeling switch reference in Koasati

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1 Introduction

Switch reference is a morphological phenomenon found in several languages that is traditionally characterized as a way of indicating whether the subjects of two conjoined clauses are the same or different (Jacobsen 1993). Examples of switch reference in Koasati, a Muskogean language spoken in Louisiana and Texas, can be seen in (1-2).[1] Here, the sentence in (1) can be followed up by (2a) or (2b). The verb at the end of the sentence in (1) has the morpheme *-k*, the same subject (SS) marker. This morpheme indicates that the subject of the following sentence will be the same as the subject of (1). (2a) and (2b) lack overt subjects, so the subject is filled in as the previously introduced subject of (1): Joe. *Ed* is marked as an object, so the interpretation of the second clause, *Edkã hihcok/hihcan*, is 'Joe saw Ed'.

(1) Joekak roomkã itcokhalihkok
Joe-k room-˜ itcokhali:ka-**k**
Joe-SBJ room-OBJ enter-**SS**
'Joe came into the room,' (Rising 1992: 4)

(2) a. Edkã hihcok cokko:lit
Ed-˜ hi:ca-**k** cokko:lit
Ed-OBJ see-**SS** sat_down
'saw Ed, and sat down.' (Rising 1992: 4)

b. Edkã hihcan cokko:lit
Ed-˜ hi:ca-**n** cokko:lit
Ed-OBJ see-**DS** sat_down
'saw Ed, and he [Ed] sat down.' (Rising 1992: 4)

In (2a), the first verb, *hihcok* ('sees'), has the morpheme *-k*, again the SS marker. This indicates that when there is no overt subject in the following clause, *cokko:lit* ('sits down'), the subject is interpreted as the subject of *hihcok*: Joe. In contrast, the first verb in (2b), *hihcan* ('sees'), has the morpheme *-n*, the different subject (DS) marker. This morpheme indicates that when there is no overt subject for *cokko:lit* ('sits down'), the subject is not interpreted as the subject of *hihcan*. Instead, it is the object, Ed.

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[1] All examples are unchanged from their sources except the third line of the gloss has been changed to use Leipzig glossing conventions.
Gloss abbreviations: 3 = THIRD PERSON; DAT = DATIVE; DIM = DIMINUTIVE; DISTR = DISTRIBUTIVE; DS = DIFFERENT SUBJECT; FOC = FOCUS; HABIT = HABITUAL; NOTHING:BUT = NOTHING BUT; OBJ = OBJECT; REALIS = REALIS; SBJ = SUBJECT; SS = SAME SUBJECT

Consider the English equivalent of (1-2) in (3). Where Koasati uses null pronouns and switch reference markers, English uses overt pronouns.

(3) Joe^{j} came into the room. He$_{j}$ saw Ed^{k}. He$_{j/k}$ sat down.

He in the third sentence could refer to either Joe or Ed. The English is ambiguous where the Koasati is not. In this paper, I account for this difference between English anaphora and Koasati switch reference using an extension of Predicate Logic with Anaphora (PLA; Dekker 1994), an ordered reference tracking system.

In addition to capturing this difference between English and Koasati, this account aims to capture the non-canonical use of switch reference in Koasati and other languages. In (4), the SS marker on *pasá:kascok* ('she seemed dirty') should indicate that *she* will again be the subject of the following sentence, but this is not the case. Instead, *she* is the indirect object of *ohimpalátka:sin* ('they were cross with her'), the verb of the following sentence, and a new subject compatible with the distributive morphology, *oh-* (DISTR), is inferred.

(4) Ho:tinannáhcok, pasá:kascok,
ho:ti-nanna-V́hco-**k** pasá:ka-:si-V́hco-**k**
sores-NOTHING:BUT-HABIT-**SS** be:dirty-DIM-HABIT-**SS**
"She was covered with sores, and she seemed dirty,"

ohimpalátka:sin. Á:yatohok,
oh-im-palátka-:si-**n** á:ya-toho-**k**
DISTR-3DAT-be:cross-DIM-**DS** go:about-REALIS-**SS**
"and people were quite cross with her. She went about,"

(Kimball 2010: 271; 68)

Syntactic accounts as in Finer 2014 cannot account for non-canonical cases of switch reference. Previous semantic analyses of switch reference include work by Stirling (1993) and McKenzie (2007, 2011, in review). Both analyze switch reference as tracking events or situations. The account in this paper will pursue a reference tracking analysis for Koasati switch reference to show that a more intuitive account, an individual reference tracking account, can capture the Koasati data. I model this data on switch reference using an extension of Predicate Logic with Anaphora, a system that maintains an ordered list of individuals in a discourse.

In Section 2, I describe Koasati switch reference. In Section 3, I introduce Dekker's (1994) Predicate Logic with Anaphora (PLA). In Section 4, I give an initial PLA analysis. In Section 5, I introduce a problem and a two-list adaptation of PLA that can solve the problem. Section 6 concludes.

2 Koasati switch reference

Koasati word order is typically SOV, with switch reference marking on the verb at the end of the clause. The verbal SS and DS morphemes are homophonous with the nominal SBJ and OBJ markings. This is summarized in Table 1. The overlap in the form of the nominal subject and object marker with the verbal switch reference

Table 1: Subject, object, and switch reference morphemes

Morpheme	Attached to Noun	Attached to Verb
-k	subject (SBJ)	same subject (SS)
-n	object (OBJ)	different subject (DS)

markers suggests that there is an important connection between nominal reference and switch reference.

(5) Joekak roomkã itcokhalihkok Edkã hihcok cokko:lit
Joe-k room-˜ itcokhali:ka-**k** Ed-˜ hi:ca-**k** cokko:lit
Joe-SBJ room-OBJ enter-**SS** Ed-OBJ see-**SS** sat_down
'Joe came into the room, saw Ed, and sat down.' (Rising 1992: 4)

Example (5) is (1) followed by (2a). A tabular breakdown of (5) is given in Table 2. Tabular breakdowns of the data will follow this pattern: the table will include only glosses of the verb/adjectives and their arguments, along with the attached switch reference marker (SR Marker). Overt arguments are in bold. In Table 2, the missing arguments are the subjects of the second and third clauses. Notice that the switch reference marker on each preceding clause is SS (same subject) and the missing arguments are filled in from the cell immediately above them.

Table 2: Breakdown of (5)

Clause	Verb Gloss	Subject	Object	SR Marker
1.	enter	**Joe**	**room**	SS
2.	see	Joe	**Ed**	SS
3.	sat_down	Joe	-	-

(6) Joekak roomkã itcokhalihkok Edkã hihcan cokko:lit
Joe-k room-˜ itcokhali:ka-**k** Ed-˜ hi:ca-**n** cokko:lit
Joe-SBJ room-OBJ enter-**SS** Ed-OBJ see-**DS** sat_down
'Joe came into the room, saw Ed, and he [Ed] sat down.' (Rising 1992: 4)

(6) is (1) followed by (2b). This is the same as (5) except that the verb in the second clause has the DS (different subject) marker instead of the SS one. The summary of (6) can be seen in Table 3. This looks the same as the previous table, except that the switch reference marker for clause 2 is DS and the missing subject of clause 3 is filled in from the cell diagonal to it (the object of clause 2), rather than the cell above it (the subject of clause 2).

Table 3: Breakdown of (6)

Clause	Verb Gloss	Subject	Object	SR Marker
1.	enter	**Joe**	**room**	SS
2.	see	Joe	**Ed**	DS
3.	sat_down	Ed	-	-

From these examples, Koasati seems to use switch reference markers to indicate which previously introduced individuals are anaphorically available to be the subject or object of a following sentence. The SS marker makes the subject and object

of the SS marked clause the available subject and object, respectively, for the next clause. The DS marker makes the subject and object of the DS marked clause the available object and subject, respectively, for the next clause. A system like PLA that can order individuals, making some more or less available for anaphora, can be used to model this data.

3 Background on PLA

Predicate Logic with Anaphora (PLA; Dekker 1994) extends standard Predicate Logic in order to keep track of individuals in a discourse. In this system there are regular truth conditions, but it also keeps track of recently introduced individuals in an information state. An example information state can be seen in (7).

(7) **A sample PLA information state**

$$s = \{\langle \underset{\uparrow \mathsf{p}_2}{a}, \underset{\uparrow \mathsf{p}_1}{b}, \underset{\uparrow \mathsf{p}_0}{c} \rangle\}$$

The information state is a set of ordered lists of individuals, corresponding to discourse referents. In PLA, the existential quantifier, $\exists$, introduces individuals into the information state. In order to pick out individuals from an information state, Dekker (1994) introduces a new term, which he calls a pronoun, that can be used to translate unbound anaphoric terms, like an English pronoun. These pronouns are given as p_i, where i is an index that refers to a particular position in a list within an information state. For example, in (7), p_0 refers to c in s, p_1 refers to b, etc. p_0 will always refer to the most recently introduced individual.

The PLA analysis of (3) shows how PLA captures the ambiguity of English pronouns. The two interpretations of (3) can be split into (8) and (9) and be translated into PLA as in Table 4 and Table 5, respectively. The difference between these two is in translating the pronoun term as p_1 or p_0.

(8) Joe$_j$ came into the room. He$_j$ saw Ed$_k$. He$_k$ sat down.
$\exists \mathsf{x}(\mathsf{x} = \mathsf{j} \wedge \exists \mathsf{y}(\mathsf{y} = \mathsf{r} \wedge \mathsf{Ixy}))\ (\wedge)\ \exists \mathsf{y}(\mathsf{y} = \mathsf{e} \wedge \mathsf{Hp_0y})\ (\wedge)\ \mathsf{Cp_0}$

(9) Joe$_j$ came into the room. He$_j$ saw Ed$_k$. He$_j$ sat down.
$\exists \mathsf{x}(\mathsf{x} = \mathsf{j} \wedge \exists \mathsf{y}(\mathsf{y} = \mathsf{r} \wedge \mathsf{Ixy}))\ (\wedge)\ \exists \mathsf{y}(\mathsf{y} = \mathsf{e} \wedge \mathsf{Hp_0y})\ (\wedge)\ \mathsf{Cp_1}$

The first clause *Joe$_j$ came into the room* is translated as $\exists \mathsf{x}(\mathsf{x} = \mathsf{j} \wedge \exists \mathsf{y}(\mathsf{y} = \mathsf{r} \wedge \mathsf{Ixy}))$. The first existential quantifier introduces the individual j (Joe), and the second introduces the individual r (the room).[2] The order of the quantifiers produces the information state with the object in the second-to-rightmost position (referred back to using p_1) and the subject in the rightmost position (referred back to using p_0). The information state update produced by the first clause can be found in the Output State column of row (b) in Tables 4-5.

[2]The room is represented as an individual to simplify the expression, but for an interpretation of *Joe$_j$ came into **a** room* a more accurate translation might be $\exists \mathsf{x}(\mathsf{x} = \mathsf{j} \wedge \exists \mathsf{y}(\mathsf{Ry} \wedge \mathsf{Ixy}))$. The resulting information state would contain a list for each of the rooms in the domain.

Table 4: Analysis of (8):

	English	PLA	Pro. Interp.	Output State
a.				$s_0 = \{\langle\rangle\}$
b.	Joe$_j$ came into the room.	$\exists x(x = j \wedge \exists y(y = r \wedge Ixy))$		$s_1 = \{\langle r, j\rangle\}$
c.	He$_j$ saw Ed$_k$.	$\exists y(y = e \wedge Hp_0 y)$	$[p_0]_{s_1} = j$	$s_2 = \{\langle r, j, e\rangle\}$
d.	He$_k$ sat down.	Cp_0	$[p_0]_{s_2} = e$	$s_3 = \{\langle r, j, e\rangle\}$

The second clause, *He$_j$ saw Ed$_k$*, can be translated into PLA as $\exists y(y = e \wedge Hp_0 y)$. The existential quantifier introduces e (Ed) into the information state. *He* is translated as p_0. When *he* is interpreted (Pro. Interp.) with respect to the input information state, s_1, it returns the individual j (Joe), the rightmost (and only) individual in the information state. The output state produced by this update is in the rightmost column of the (c) row in each table.

The final clause captures the ambiguity of the English pronoun. *He$_{j/k}$ sat down* can be translated as Cp_0, as in row (d) of Table 4, or as Cp_0, as in (d) of Table 5.

Table 5: Analysis of (9):

	English	PLA	Pro. Interpr.	Output State
a.				$s_0 = \{\langle\rangle\}$
b.	Joe$_j$ came into the room.	$\exists x(x = j \wedge \exists y(y = r \wedge Ixy))$		$s_1 = \{\langle r, j\rangle\}$
c.	He$_j$ saw Ed$_k$.	$\exists y(y = e \wedge Hp_0 y)$	$[p_0]_{s_1} = j$	$s_2 = \{\langle r, j, e\rangle\}$
d.	He$_j$ sat down.	Cp_1	$[p_1]_{s_2} = j$	$s_3 = \{\langle r, j, e\rangle\}$

In English, the ambiguity of *he* is represented in PLA by different pronoun terms: p_0 and p_1, referring to Ed (more recent) or Joe (less recent).

4 PLA analysis

The lack of ambiguity in the Koasati data can be captured by translating the subject agreement marker as p_0 and object agreement marker as p_1. Further, the switch reference markers can be translated so that the DS marker swaps the order of the individuals in the p_0 and p_1 positions and the SS marker maintains the order. The translations of Koasati morphemes are summarized below. The SBJ and OBJ markers manipulate the information state in a similar way to the SS and DS markers. The main difference between the two is that the SBJ/OBJ markers introduce new individuals to the information state, and the SS/DS markers only manipulate the ones already in the information state.

(10) **Koasati morphemes translated into PLA**

a. a-SBJ: $\exists z(z = a)$
b. b-OBJ: $\exists x(x = p_0 \wedge \exists z(z = b))$
c. intransitive verb: Vp_0
d. SS: $\exists x(x = p_0 \wedge \exists y(y = p_1))$
e. DS: $\exists y(y = p_1 \wedge \exists x(x = p_0))$
f. transitive verb: Vp_0p_1

The effect of the SS and DS markers on the information state can be seen in (11-12), respectively. In (11), the SS marker copies the two rightmost individuals of

the information state to the end of the list in the same order to produce the output state. In (12), the DS marker also copies the two rightmost individuals to the end of the information state, but it puts them in the opposite order. Thus, the previous available subject and object become the available object and subject, respectively, for the following sentence.

(11) **SS marker**

$$s_n = \{\langle \quad a, \quad b, \quad c \quad \rangle\} \overset{\text{SS}}{\rightarrow} \quad s_{n+1} = \{\langle \quad \langle a, \quad b, \quad c, \quad b, \quad c \quad \rangle\}$$

(12) **DS marker**

$$s_n = \{\langle \quad a, \quad b, \quad c \quad \rangle\} \overset{\text{DS}}{\rightarrow} \quad s_{n+1} = \{\langle \quad \langle a, \quad b, \quad c, \quad c, \quad b \quad \rangle\}$$

(1) Joekak roomkã itcokhalihkok
Joe-k room-˜ itcokhali:ka-**k**
Joe-SBJ room-OBJ enter-**SS**
'Joe came into the room,' (Rising 1992: 4)

The data from (1) is repeated here. The translation of (1) into PLA can be seen in Table 6. The nouns marked as subjects and objects are translated to be introduced separately from the verb so that a newly introduced object doesn't displace the subject.[3] The effect is that the subject is the rightmost individual and the object is the second-to-rightmost individual. Thus, in (a) and (b) of Table 6, j and r are added to the information state in the correct order. Update with (c) would remove any lists in the information state where the I (enter) relationship did not hold between the two rightmost individuals of the information state. The SS marker in (d) copies r and j to the end of the output information state, in the same order.

Table 6: Analysis of (1)

	Gloss	PLA	Pro. Interp.	Output State
a.	Joe-SBJ	$\exists z(z = j)$		$s_1 = \{\langle j \rangle\}$
b.	room-OBJ	$\exists x(x = p_0 \wedge \exists z(z = r))$	$[p_0]_{s_1} = j$	$s_2 = \{\langle j, r, j \rangle\}$
c.	enter	$I p_0 p_1$	$[p_1]_{s_2} = r$, $[p_0]_{s_2} = j$	$s_3 = \{\langle j, r, j \rangle\}$
d.	-SS	$\exists x(x = p_0 \wedge \exists y(y = p_1))$	$[p_1]_{s_3} = r$, $[p_0]_{s_3} = j$	$s_4 = \{\langle j, r, j, r, j \rangle\}$

Tables 7 and 8 are possible continuations of Table 6 that correspond to the follow up sentences of (2a) and (2b), respectively.

[3] An alternative way of translating Koasati verbs would have existential quantifiers quantifying over variables that are part of the predicate. Working this way, (1), excluding the switch reference marker, would be translated as $\exists x(x = j \wedge \exists z(z = r \wedge Ixz))$. This sort of translation would require that each argument be quantified over by an existential quantifier to maintain the correct order of subject and object individuals. It is also not compositional since the SR marker is not included.

(2a) Edkã hihcok cokko:lit
Ed-˜ hi:ca-**k** cokko:lit
Ed-OBJ see-**SS** sat_down
'saw Ed, and sat down.'
(Rising 1992: 4)

(2b) Edkã hihcan cokko:lit
Ed-˜ hi:ca-**n** cokko:lit
Ed-OBJ see-**DS** sat_down
'saw Ed, and he [Ed] sat down.'
(Rising 1992: 4)

In (e) of Table 7, e (Ed) is added to the information state in the object (second-to-rightmost) position. Then update with (f) would filter out any lists where the H (see) relationship did not hold between the right two individuals. Update with the SS marker of (g) copies the two rightmost individuals in input state, s_6. Then, when s_7 is updated with $\mathsf{Cp_o}$ (sat_down), the pronoun p_o is interpreted as the rightmost individual of input information state s_7: j (Joe). This gives the interpretation that Joe sat down.

Table 7: Analysis of (2a)

	Gloss	PLA	Pro. Interp.	Output State
e.	Ed-OBJ	$\exists \mathsf{x}(\mathsf{x} = \mathsf{p_o} \wedge \exists \mathsf{z}(\mathsf{z} = \mathsf{e}))$	$[\mathsf{p_o}]_{s_4} = j$	$s_5 = \{\langle j, r, j, r, j, e, j\rangle\}$
f.	see	$\mathsf{Hp_o p_1}$	$[\mathsf{p_1}]_{s_5} = e$, $[\mathsf{p_o}]_{s_5} = j$	$s_6 = \{\langle j, r, j, r, j, e, j\rangle\}$
g.	-**SS**	$\exists \mathsf{x}(\mathsf{x} = \mathsf{p_o} \wedge \exists \mathsf{y}(\mathsf{y} = \mathsf{p_1}))$	$[\mathsf{p_1}]_{s_6} = e$, $[\mathsf{p_o}]_{s_6} = j$	$s_7 = \{\langle j, r, j, r, j, e, j, e, j\rangle\}$
h.	sat_down	$\mathsf{Cp_o}$	$[\mathsf{p_o}]_{s_7} = j$	$s_8 = \{\langle j, r, j, r, j, e, j, e, j\rangle\}$

The PLA translation of (2b) in Table 8 procedes just like the translation of (2a) through row (f). In row (g), the information state, s_6 is updated with DS instead of SS. This copies the two rightmost individuals of s_6 to the output state s_7 in the opposite order. Then when s_7 is updated with $\mathsf{Cp_o}$ in (h), the rightmost individual of the input information state is e (Ed) instead of j (Joe). Thus, the interpretation of (2a) is that Ed sat down.

Table 8: Analysis of (2b)

	Gloss	PLA	Pronoun Interp.	Output State
e.	Ed-OBJ	$\exists \mathsf{x}(\mathsf{x} = \mathsf{p_o} \wedge \exists \mathsf{z}(\mathsf{z} = \mathsf{e}))$	$[\mathsf{p_o}]_{s_4} = j$	$s_5 = \{\langle j, r, j, r, j, e, j\rangle\}$
f.	see	$\mathsf{Hp_o p_1}$	$[\mathsf{p_1}]_{s_5} = e$, $[\mathsf{p_o}]_{s_5} = j$	$s_6 = \{\langle j, r, j, r, j, e, j\rangle\}$
g.	-**DS**	$\exists \mathsf{y}(\mathsf{y} = \mathsf{p_1} \wedge \exists \mathsf{x}(\mathsf{x} = \mathsf{p_o}))$	$[\mathsf{p_1}]_{s_6} = e$, $[\mathsf{p_o}]_{s_6} = j$	$s_7 = \{\langle j, r, j, r, j, e, j, j, e\rangle\}$
h.	sat_down	$\mathsf{Cp_o}$	$[\mathsf{p_o}]_{s_7} = e$	$s_8 = \{\langle j, r, j, r, j, e, j, j, e\rangle\}$

In the (g) rows of Tables 7 and 8, the different, information-state-altering translations for the switch reference morphemes generate distinct, unambiguous interpretations for the Koasati examples. This use of PLA takes advantage of PLA's prominence based tracking designed for English anaphora and extends it to Koasati

anaphora by basing the prominence of elements on the role of the noun in the sentence rather than on recency. It seems like it is possible to extend a system like PLA to account for cross-linguistic anaphora based on different types of prominence. This raises the question of whether we can describe all systems of anaphora using a prominence based system that varies in what factors determine prominence.

5 A problem and a proposed solution: The two list analysis

It is also possible to follow up the sentence in (13) with (14). This cannot be accounted for using the system discussed above.

(13) Joekak roomkã itcokhali:kon
Joe-k room-˜ itcokhali:ka-**n**
Joe-SBJ room-OBJ enter-**DS**
'Joe came into the room,' (Rising 1992: 4)

(14) Edkak hihcan cokko:lit
Ed-k hi:ca-**n** cokko:lit
Ed-SBJ see-**DS** sat_down
'Ed saw him, and Joe sat down.' (Rising 1992: 4)

The problem is not with the generalization from earlier. As can be seen in Table 9, the argument in clause 3 that is missing is filled in from the diagonal cell (the object of clause 2) when clause 2 has a DS morpheme.

Table 9: Breakdown of (13)

Clause	Verb Gloss	Subject	Object	SR Marker
1.	enter	**Joe**	**room**	DS
2.	see	**Ed**	Joe	DS
3.	sat_down	Joe	-	-

The problem comes from the way that the DS marker is translated. As can be seen in Table 10, the PLA translation of (13), the DS marker in row (d) copies the two rightmost individuals r and j into the output state in the opposite order. This puts r in the subject position, the rightmost position on the list.

Table 10: Analysis of (13)

	Gloss	PLA	Pronoun Interp.	Output State
a.	Joe-SBJ	$\exists z(z = j)$		$s_1 = \{\langle j \rangle\}$
b.	room-OBJ	$\exists x(x = p_0 \wedge \exists z(z = r))$	$[p_0]_{s_1} = j$	$s_2 = \{\langle j, r, j \rangle\}$
c.	enter	lp_0p_1	$[p_1]_{s_2} = r$, $[p_0]_{s_2} = j$	$s_3 = \{\langle j, r, j \rangle\}$
d.	-**DS**	$\exists y(y = p_1 \wedge \exists x(x = p_0))$	$[p_1]_{s_3} = r$, $[p_0]_{s_3} = j$	$s_4 = \{\langle j, r, j, j, r \rangle\}$

When the translation of (13) in Table 10 is followed up by the translation of (14) in Table 11, the input information state is already wrong in that r is the available

subject, not j. Then, update with (e) adds e (Ed) to the subject position of the information state, so j is displaced by r in the object position of the information state. Update with $\mathsf{Hp_0p_1}$ in row (f) gets the interpretation that Ed saw the room. Update with DS in (g) then switches the order of e and r in the information state, so update with $\mathsf{Cp_0}$ in row (h) gets the interpretation that the room sat down.

Table 11: Analysis of (14)

	Gloss	PLA	Pronoun Interp.	Output State
e.	Ed-SBJ	$\exists \mathsf{x}(\mathsf{x} = \mathsf{e})$		$s_5 = \{\langle j, r, j, j, \mathbf{r}, e\rangle\}$
f.	see	$\mathsf{Hp_0p_1}$	$[\mathsf{p_1}]_{s_5} = \mathbf{r}$,$[\mathsf{p_0}]_{s_5} = e$	$s_6 = \{\langle j, r, j, j, \mathbf{r}, e, r\rangle\}$
g.	**-DS**	$\exists \mathsf{y}(\mathsf{y} = \mathsf{p_1} \wedge \exists \mathsf{x}(\mathsf{x} = \mathsf{p_0}))$	$[\mathsf{p_1}]_{s_6} = e$,$[\mathsf{p_0}]_{s_6} = \mathbf{r}$	$s_7 = \{\langle j, r, j, j, r, e, e, \mathbf{r}\rangle\}$
h.	sat_down	$\mathsf{Cp_0}$	$[\mathsf{p_0}]_{s_7} = \mathbf{r}$	$s_8 = \{\langle j, r, j, j, r, e, e, \mathbf{r}\rangle\}$

The issue here is that this system doesn't quite reproduce the pattern described in the previous section where the subject of the DS marked clause becomes the available object of the following clause. This is a problem that can potentially be fixed by separating the subjects from the objects in the information state.

5.1 Two list analysis: Two List PLA (TLPLA) for Switch Reference

Separating the subjects and the objects can be done by having two lists of individuals instead of one. Such a system has been proposed by Bittner (2001, 2011) to account for anaphora and for the obviative system in Kalallisut (West Greenlandic), among other phenomena. The intuition is that PLA's prominence based anaphora can be divided into foregrounded ($\top$) and backgrounded ($\bot$) individuals. Little & Moroney (2016) use a variation of this two list system (Two List PLA; TLPLA) in an analysis of obviation in Mi'gmaq, where discourse salience divides individuals into the foreground and background. Here, I use a variation of the TLPLA system. An example of a two list information state can be seen in (15).

(15) **A sample two list information state**

$$s = \{\langle \quad \underset{\mathsf{p}^{\top}{}_1}{\langle a,} \quad \underset{\mathsf{p}^{\top}{}_0}{b\rangle,} \quad \underset{\mathsf{p}^{\bot}{}_1}{\langle c} \quad \underset{\mathsf{p}^{\bot}{}_0}{d\rangle} \quad \rangle\}$$

In (15), instead of an information states being a set of ordered lists of individuals, it is a set of pairs of ordered lists. There are separate lists used for individuals introduced in the subject position and those introduced in the object position. The subject list is the list on the left, and the pronouns that are used to pick out individuals on the list have the superscript $\top$. The object list is on the right, and the pronouns used to pick out individuals from that list have the superscript $\bot$. The indices still refer to the position of the individual on the subject or object list. For example, $\mathsf{p}^{\top}{}_1$ refers to the second-to-rightmost individual on the subject list: a. The idea for Koasati is that the subject individuals are foregrounded, the object individuals are backgrounded, and the switch reference morphemes can manipulate which previously introduced individuals are in the foreground or background. The two list PLA translations for the relevant Koasati morphemes can be seen below.

(16) **Koasati morphemes translated into TLPLA**

a. a-SBJ: $\exists^{\top}\mathsf{z}(\mathsf{z}=\mathsf{a})$
b. b-OBJ: $\exists^{\perp}\mathsf{z}(\mathsf{z}=\mathsf{b})$
c. intransitive verb: $\mathsf{Vp}^{\top}{}_{\mathrm{o}}$
d. SS: $\exists_{\perp}\mathsf{x}(\mathsf{x}=\mathsf{p}^{\perp}{}_{\mathrm{o}}\wedge\exists_{\perp}\mathsf{y}(\mathsf{y}=\mathsf{p}^{\top}{}_{\mathrm{o}})$
e. DS: $\exists^{\top}\mathsf{y}(\mathsf{y}=\mathsf{p}^{\perp}{}_{\mathrm{o}})\wedge\exists_{\perp}\mathsf{x}(\mathsf{x}=\mathsf{p}^{\top}{}_{\mathrm{o}})$
f. transitive verb: $\mathsf{Vp}^{\top}{}_{\mathrm{o}}\mathsf{p}^{\perp}{}_{\mathrm{o}}$

The SS and DS markers now alter the information state in a different way. In (17), the SS marker copies the rightmost individual of the object list and the rightmost individual of the subject list to the object list of the output state. This allows there to be an individual on the object list when no object has been introduced. In (18), the DS marker copies the individual from the subject list to the object list and the object list to the subject list.

(17) **SS marker**

$$s_n = \{\langle\, \langle a,\ b\rangle,\ \langle c,\ d\rangle\,\rangle\} \overset{\text{SS}}{\rightarrow} s_{n+1} = \{\langle\, \langle a,\ b\rangle,\ \langle c,\ d,\ b,\ d\rangle\,\rangle\}$$

(18) **DS marker**

$$s_n = \{\langle\, \langle a,\ b\rangle,\ \langle c,\ d\rangle\,\rangle\} \overset{\text{DS}}{\rightarrow} s_{n+1} = \{\langle\, \langle a,\ b,\ d\rangle,\ \langle c,\ d,\ b\rangle\,\rangle\}$$

5.2 Accounting for problematic data in (13)

The issue introduced at the beginning of section 5 was that the one list system was not capturing the generalization that the subject of a DS marked clause becomes the available object of the following clause. The TLPLA system can account for the problematic data by keeping the available subject and object individuals separate. Table 12 is the PLA translation of (13). In row (a) and (b) of this table, j and r are added to the subject and object list, respectively. Update with (c) would filter out any lists in the input state that do not have an I (enter) relationship between the rightmost subject and object list individuals. In (d), the DS markers copies j to the object list and r to the subject list.

Table 12: Analysis of (13)

	Gloss	PLA	Pronoun Interp.	Output State
a.	Joe-SBJ	$\exists\mathsf{z}(\mathsf{z}=\mathsf{j})$		$s_1 = \{\langle\langle j\rangle,\langle\rangle\rangle\}$
b.	room-OBJ	$\exists^{\perp}\mathsf{z}(\mathsf{z}=\mathsf{r})$		$s_2 = \{\langle\langle j\rangle,\langle r\rangle\rangle\}$
c.	enter	$\mathsf{Ip}^{\top}{}_{\mathrm{o}}\mathsf{p}^{\perp}{}_{\mathrm{o}}$	$[\mathsf{p}^{\top}{}_{\mathrm{o}}]_{s_2} = j$, $[\mathsf{p}^{\perp}{}_{\mathrm{o}}]_{s_2} = r$	$s_3 = \{\langle\langle j\rangle,\langle r\rangle\rangle\}$
d.	-DS	$\exists\mathsf{y}(\mathsf{y}=\mathsf{p}^{\perp}{}_{\mathrm{o}})\wedge$ $\exists^{\perp}\mathsf{x}(\mathsf{x}=\mathsf{p}^{\top}{}_{\mathrm{o}})$	$[\mathsf{p}^{\perp}{}_{\mathrm{o}}]_{s_3} = r$, $[\mathsf{p}^{\top}{}_{\mathrm{o}}]_{s_3} = j$	$s_4 = \{\langle\langle j,r\rangle,\langle r,j\rangle\rangle\}$

Table 13 is a PLA translation of (14). In (e), ed (Ed) is added to the subject list. Note that j is still the available object, which fits the data. In (f), $\mathsf{Hp}^{\top}{}_{\mathrm{o}}\mathsf{p}^{\perp}{}_{\mathrm{o}}$ filters out any lists where a H (see) relationship doesn't hold between the subject and object.

Table 13: Analysis of (14)

	Gloss	PLA	Pronoun Interp.	Output State
e.	Ed-SBJ	$\exists \mathsf{z}(\mathsf{z} = \mathsf{e})$		$s_5 = \{\langle\langle j, r, e\rangle, \langle r, j\rangle\rangle\}$
f.	see	$\mathsf{Hp}^{\top}{}_{\mathrm{o}}\mathsf{p}^{\bot}{}_{\mathrm{o}}$	$[\mathsf{p}^{\top}{}_{\mathrm{o}}]_{s_5} = e$, $[\mathsf{p}^{\bot}{}_{\mathrm{o}}]_{s_5} = j$	$s_6 = \{\langle\langle j, r, e\rangle, \langle r, j\rangle\rangle\}$
g.	-**DS**	$\exists \mathsf{y}(\mathsf{y} = \mathsf{p}^{\bot}{}_{\mathrm{o}}) \wedge \exists^{\bot}\mathsf{x}(\mathsf{x} = \mathsf{p}^{\top}{}_{\mathrm{o}})$	$[\mathsf{p}^{\top}{}_{\mathrm{o}}]_{s_6} = e$, $[\mathsf{p}^{\bot}{}_{\mathrm{o}}]_{s_6} = j$	$s_7 = \{\langle\langle j, r, e, j\rangle, \langle r, j, e\rangle\rangle\}$
h.	sat_down	$\mathsf{Cp}^{\top}{}_{\mathrm{o}}$	$[\mathsf{p}^{\top}{}_{\mathrm{o}}]_{s_7} = j$	$s_8 = \{\langle\langle j, r, e, j\rangle, \langle r, j, e\rangle\rangle\}$

The DS marker in (g) makes j the available subject and e the available object. Then in (h), $\mathsf{Cp}^{\top}{}_{\mathrm{o}}$ gets the interpretation that j (Joe) sat down.

This two list adaptation of PLA seems to be able to account for the data. In Appendix A, you can see the two list PLA translations of examples (1), (2a), and (2b) from above.

5.3 Examples from texts

Examples (19-22) are part of a story from a book of Koasati narratives (Kimball 2010). Under each glossed example are the information states as they are updated by words and morphemes. Both the one list and two list systems are shown. Due to space limitations, only the updates that alter the information state are given.

In (19), the empty information state is first updated with WOMAN, which adds w to the list in the one list system and adds w to the subject list in the two list system.[4] Update with the verb doesn't change the information state in this simplified case, so it is not shown here. Update with the SS marker copies w to the end of the list in the one list system and copies w to the object list in the two list system.

(19) Tayyí sihnóhcok á:yatoho:limpatš
tayyí sihno-Vhco-**k** á:ya-toho-:li-mpa-t-š
woman old-HABIT-**SS** go:about-REALIS-DEDUC-HEARSAY-PAST-PH:TR
"It is said that an elderly woman was going about."

(Kimball 2010: 271; 68)

One list:	$\langle\rangle$	$\overset{\text{WOMAN}}{\rightarrow}$	$\langle w\rangle$	$\overset{\text{SS}}{\rightarrow}$	$\langle w, w\rangle$
Two list:	$\langle\langle\rangle, \langle\rangle\rangle$	$\overset{\text{WOMAN}}{\rightarrow}$	$\langle\langle w\rangle, \langle\rangle\rangle$	$\overset{\text{SS}}{\rightarrow}$	$\langle\langle w\rangle, \langle w\rangle\rangle$

(20) has more verbs with SS markers that copy parts of the information state the same way as the previous example.

(20) Ho:tinannáhcok, pasá:kascok,
ho:ti-nanna-Vhco-**k** pasá:ka-:si-Vhco-**k**
sores-NOTHING:BUT-HABIT-**SS** be:dirty-DIM-HABIT-**SS**
"She was covered with sores, and she seemed dirty,"

(Kimball 2010: 271; 68)

[4] As with the PLA interpretation of 'room' above, 'woman' is being treated as introducing only one individual to simplify the information state.

One list:	$\langle w, w\rangle$	$\overset{\text{SS}}{\rightarrow}$	$\langle w, w, w\rangle$	$\overset{\text{SS}}{\rightarrow}$	$\langle w, w, w, w\rangle$
Two list:	$\langle\langle w\rangle, \langle w\rangle\rangle$	$\overset{\text{SS}}{\rightarrow}$	$\langle\langle w\rangle, \langle w, w\rangle\rangle$	$\overset{\text{SS}}{\rightarrow}$	$\langle\langle w\rangle, \langle w, w, w\rangle\rangle$

(20-21), repeated from (4), show that both the one and two list system can account for this instance of non-canonical switch reference. Here, *pasá:kascok* ('she seemed dirty') in (20), ends with the SS marker. Canonically, the expectation would be that the following clause should have 'the woman' as the subject. Instead, 'the woman' is the indirect object, cross-referenced by the morpheme *im-* in *ohimpalátka:sin* ('people were quite cross with her'). Then, where does the new subject come from? This is the beginning of the story, so the 'people' have not been mentioned before. It can only come from the distributive morphology *oh-* that is incompatible with the features of the only individual that is available in the input information state. Continuing on, *ohimpalátka:sin* has the DS morpheme. In the one list system, this moves the second-to-rightmost individual to the rightmost position and vice versa. In the two list system the individual at the end of the subject list is copied to the object list and the individuals at the end of the object list is copied to the subject list. The effect is the same: w is the available subject and p is the available object.

(21) ohimpalátka:sin. Á:yatohok,
oh-im-palátka-:si-**n** á:ya-toho-**k**
DISTR-3DAT-be:cross-DIM-**DS** go:about-REALIS-**SS**
"and people were quite cross with her. She went about,"
(Kimball 2010: 271; 68)

One list:	$\langle \ldots, w\rangle$	$\overset{\text{DISTR}}{\rightarrow}$	$\langle \ldots, w, p\rangle$	$\overset{\text{DS}}{\rightarrow}$	$\langle \ldots, w, p, w\rangle$
Two list:	$\langle\langle w\rangle, \langle w, w, w\rangle\rangle$	$\overset{\text{DISTR}}{\rightarrow}$	$\langle\langle w, p\rangle, \langle w, w, w\rangle\rangle$	$\overset{\text{DS}}{\rightarrow}$	$\langle\langle w, p, w\rangle, \langle w, w, w, p\rangle\rangle$

(22) similarly show that in addition to overt nouns, number morphology must be able to add individuals to the lists. Here, again, given the SS marker on the previous clause *Á:yatohok* ('she went about'), we would expect the subject of the following clause to be 'the woman'. Instead, a new subject is introduced. The form of the atlawístanannáhcok is marked for plural, which is not compatible with the available subject 'the woman'. This introduces a new subject into the information state: c.

(22) atlawístanannáhcok,
at-lawísta-nanna-V́hco-**k**
person-small(pl)-NOTHING:BUT-HABIT-**SS**
"and there were nothing but children; " (Kimball 2010: 271; 68)

One list:	$\langle \ldots, w, p, w\rangle$ $\langle \ldots, w, p, w, c, w, c\rangle$	$\overset{\text{PERSON-SMALL(PL)}}{\rightarrow}$	$\langle \ldots, w, p, w, c\rangle$	$\overset{\text{SS}}{\rightarrow}$
Two list:	$\langle\langle w, p, w\rangle, \langle w, w, w, p\rangle\rangle$ $\langle\langle w, p, w, c\rangle, \langle \ldots, p, c, p\rangle\rangle$	$\overset{\text{PERSON-SMALL(PL)}}{\rightarrow}$	$\langle\langle w, p, w, c\rangle, \langle \ldots, p\rangle\rangle$	$\overset{\text{SS}}{\rightarrow}$

Table 14 gives the tabular breakdown of (19-22). The patterns for non-overt arguments is consistent with the patterns previously shown except that in some

cases, morphology on a verb/adjective acts as the overt element that introduces a new individual into the discourse.

Table 14: Breakdown of (19-22)

Ex.	Clause	Verb Gloss	Subject	Object	SR Marker
(19)	1	old	**woman**		SS
(19)	2	go_about	woman		PH:TERM
(20)	1	sores-NOTHING:BUT	woman	-	SS
(20)	2	be:dirty	woman	-	SS
(21)	1	DISTR-3DAT-be:cross	**DISTR**:people	woman	DS
(21)	2	go:about	woman	-	SS
(22)	1	person-small(pl)	**PL**:children	-	SS

These examples from the narrative text show that the generalizations made about Koasati switch reference are compatible with what has previously been covered in this paper. Additionally, it shows that verbal/adjectival morphology plays a role in adding individuals to an information state.

6 Conclusion

I have presented basic data of switch reference in Koasati and discussed two semantic analyses for this data. The goal of the approach proposed here is to account for switch reference in the nominal domain as opposed to an analysis based on events or situations. The first proposed analysis uses Predicate Logic with Anaphora (Dekker 1994), a system of prominence based anaphora that stores a list of individuals introduced in discourse. The second analysis discussed modifies PLA to a two list system, adapting Bittner (2001, 2011), using the TLPLA analysis of Little & Moroney (2016) to separate subjects and objects into separate lists. Both accounts can make the correct predictions for non-canonical cases of switch reference. For this analysis, it seems to be necessary for verbal morphology to be able to add individuals to the information state. It seems that a reference tracking analysis in the nominal domain can do a lot of work to account for Koasati switch reference, which is nice given the connection between verbal switch reference marking and nominal subject/object marking. According to Bittner "the PLA theory of prominence-based discourse anaphora is refined to distinguish discourse entities that are currently in the center of attention versus background" (2014: 6). The implementation of PLA for English relies on recency to determine prominence. For obviation marking languages, there is a grammatical marking to indicate an individual is more or less prominent in the discourse (Bittner 2011; Little & Moroney 2016). For Koasati, prominence seems to be related to grammatical role in the sentence—i.e., whether the individual is introduced as a subject or not. It would be interesting to see how prominence is marked and how it plays a role in anaphora cross-linguistically.

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A TLPLA analyses of extra data

Tables 15-17 show that the two list version of PLA can still account for the initial data from (1), (2a), and (2b).

Table 15: TLPLA analysis of (1)

	Gloss	PLA	Pronoun Interp.	Output State
a.	Joe-SBJ	$\exists^{\top}\mathsf{z}(\mathsf{z}=\mathsf{j})$		$s_1 = \{\langle\langle j\rangle, \langle\rangle\rangle\}$
b.	room-OBJ	$\exists^{\bot}\mathsf{z}(\mathsf{z}=\mathsf{r})$		$s_2 = \{\langle\langle j\rangle, \langle r\rangle\rangle\}$
c.	enter	$\mathsf{l}\mathsf{p}^{\top}{}_{\mathrm{o}}\mathsf{p}^{\bot}{}_{\mathrm{o}}$	$[\mathsf{p}^{\top}{}_{\mathrm{o}}]_{s_2} = j$, $[\mathsf{p}^{\bot}{}_{\mathrm{o}}]_{s_2} = r$	$s_3 = \{\langle\langle j\rangle, \langle r\rangle\rangle\}$
d.	-SS	$\exists^{\bot}\mathsf{x}(\mathsf{x}=\mathsf{p}^{\bot}{}_{\mathrm{o}} \wedge \exists^{\bot}\mathsf{y}(\mathsf{y}=\mathsf{p}^{\top}{}_{\mathrm{o}}))$	$[\mathsf{p}^{\bot}{}_{\mathrm{o}}]_{s_3} = r$, $[\mathsf{p}^{\top}{}_{\mathrm{o}}]_{s_3} = j$	$s_4 = \{\langle\langle j\rangle, \langle r, j, r\rangle\rangle\}$

Table 16: TLPLA analysis of (2a)

	Gloss	PLA	Pronoun Interp.	Output State
e.	Ed-OBJ	$\exists^{\bot}\mathsf{z}(\mathsf{z}=\mathsf{e})$		$s_5 = \{\langle\langle j\rangle, \langle r, j, r, e\rangle\rangle\}$
f.	see	$\mathsf{H}\mathsf{p}^{\top}{}_{\mathrm{o}}\mathsf{p}^{\bot}{}_{\mathrm{o}}$	$[\mathsf{p}^{\top}{}_{\mathrm{o}}]_{s_5} = j$, $[\mathsf{p}^{\bot}{}_{\mathrm{o}}]_{s_5} = e$	$s_6 = \{\langle\langle j\rangle, \langle r, j, r, e\rangle\rangle\}$
g.	-SS	$\exists^{\bot}\mathsf{x}(\mathsf{x}=\mathsf{p}^{\bot}{}_{\mathrm{o}} \wedge \exists^{\bot}\mathsf{y}(\mathsf{y}=\mathsf{p}^{\top}{}_{\mathrm{o}}))$	$[\mathsf{p}^{\top}{}_{\mathrm{o}}]_{s_6} = j$, $[\mathsf{p}^{\bot}{}_{\mathrm{o}}]_{s_6} = e$	$s_7 = \{\langle\langle j\rangle, \langle r, j, r, e, j, e\rangle\rangle\}$
h.	sat_down	$\mathsf{C}\mathsf{p}^{\top}{}_{\mathrm{o}}$	$[\mathsf{p}^{\top}{}_{\mathrm{o}}]_{s_7} = j$	$s_8 = \{\langle\langle j\rangle, \langle r, j, r, e, j, e\rangle\rangle\}$

Table 17: TLPLA analysis of (2b)

	Gloss	PLA	Pronoun Interp.	Output State
e.	Ed-OBJ	$\exists^{\perp}\mathsf{z}(\mathsf{z}=\mathsf{e})$		$s_5 = \{\langle\langle j\rangle, \langle r, j, r, e\rangle\rangle\}$
f.	see	$\mathsf{H}\mathsf{p}^{\top}{}_{\mathrm{o}}\mathsf{p}^{\perp}{}_{\mathrm{o}}$	$[\mathsf{p}^{\top}{}_{\mathrm{o}}]_{s_5} = j$, $[\mathsf{p}^{\perp}{}_{\mathrm{o}}]_{s_5} = e$	$s_6 = \{\langle\langle j\rangle, \langle r, j, r, e\rangle\rangle\}$
g.	**-DS**	$\exists\mathsf{y}(\mathsf{y}=\mathsf{p}^{\perp}{}_{\mathrm{o}}) \wedge \exists^{\perp}\mathsf{x}(\mathsf{x}=\mathsf{p}^{\top}{}_{\mathrm{o}})$	$[\mathsf{p}^{\top}{}_{\mathrm{o}}]_{s_6} = j$, $[\mathsf{p}^{\perp}{}_{\mathrm{o}}]_{s_6} = e$	$s_7 = \{\langle\langle j, e\rangle, \langle r, j, r, e, j\rangle\rangle\}$
h.	sat_down	$\mathsf{C}\mathsf{p}^{\top}{}_{\mathrm{o}}$	$[\mathsf{p}^{\top}{}_{\mathrm{o}}]_{s_7} = e$	$s_8 = \{\langle\langle j, e\rangle, \langle r, j, r, e, j\rangle\rangle\}$

B TLPLA for Switch Reference

TLPLA for switch reference is very similar to the TLPLA system described in Little & Moroney 2016. This section includes only the differences between TLPLA for switch reference and that system. In **DEFINITION 2.1**, the definition of the information states is the same except that the TLPLA in Little & Moroney 2016 also defines the top and bottom list—$l^{\top}$ and $l^{\perp}$, respectively—that form a case e.

DEFINITION 2.1 (Information States)

$*$3. For a state $s \in S^n$, where $a + b = n$ and $0 < j \leq a$, and for any case $e = \langle\langle {d^{\top}}_1, \ldots, {d^{\top}}_a\rangle, \langle {d^{\perp}}_1, \ldots, {d^{\perp}}_b\rangle\rangle \in s$, ${d^{\top}}_j$ is a possible value for the j-th subject of s, also indicated as ${e^{\top}}_j$.

4. For a state $s \in S^n$, where $a + b = n$ and $0 < k \leq b$, and for any case $e = \langle\langle {d^{\top}}_1, \ldots, {d^{\top}}_a\rangle, \langle {d^{\perp}}_1, \ldots, {d^{\perp}}_b\rangle\rangle \in s$, ${d^{\perp}}_k$ is a possible value for the k-th subject of s, also indicated as ${e^{\perp}}_k$.

DEFINITION 2.2 is different in that the definition of extension requires that some case in the input state survive with the same top ($\top$) and bottom ($\perp$) list as a subpart of the new case in the output state. The definition in Little & Moroney 2016 uses a definition of extension that allows a case to be an extension if the top and bottom lists that make up the input case survive as a part of the case in the output state even if they are no longer still in their original top or bottom list. This was necessary because the definition of $\exists$ in Little & Moroney 2016 could move a list from being the top list to being part of the bottom list or vice versa. In contrast, the $\exists$ in **DEFINITION 3.2** here simply adds an individual to the relevant list.

DEFINITION 2.2 (Notational Convention)

1. If $e \in D^n$ and $e' \in D^m$, then $e \cdot e' = \langle e_1, \ldots e_n, {e'}_1, \ldots, {e'}_m\rangle \in D^{n+m}$
2. e' is an extension of e, $e \leq e'$, iff $\exists e'' : e' = e \cdot e''$
3. e' is an extension of e, $e \leq e'$, iff $\forall {e^{\top}}' \in e' \; \exists e^{\top} \in e : e^{\top} \leq {e^{\top}}' \;\&\; \forall {e^{\perp}}' \in e' \exists e^{\perp} \in e : e^{\perp} \leq {e^{\perp}}'$
4. For $s \in S^n (i \in D^n)$, $N_s = n (= a + b)$, $N_{\top} = a$, $N_{\perp} = b$, the number of subjects of $s(i)$

DEFINITION 3.2 (Semantics of PLA)

4. $s[\![\exists^{\top} x\phi]\!]_{\mathcal{M},g} = \{\langle e^{\top} \cdot d, e^{\perp}\rangle \mid d \in D \;\&\; \langle e^{\top}, e^{\perp}\rangle \in s[\![\phi]\!]_{\mathcal{M},g[x/d]}\}$
5. $s[\![\exists^{\perp} x\phi]\!]_{\mathcal{M},g} = \{\langle e^{\top}, e^{\perp} \cdot d\rangle \mid d \in D \;\&\; \langle e^{\top}, e^{\perp}\rangle \in s[\![\phi]\!]_{\mathcal{M},g[x/d]}\}$